Homework #7, due Tuesday, June 2nd.

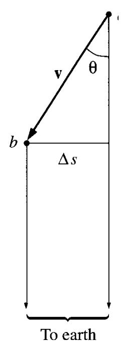


Figure 12.6

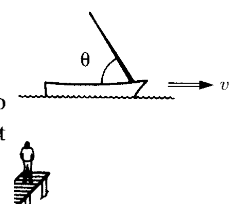
Problem X: (A) A photon with wavelength λ_1 scatters off an electron (mass m), which was initially at rest. Derive the formula for Compton scattering, relating the wavelength of the reflected photon λ_2 to λ_1 and the scattering angle θ (angle between direction of propagation of the incident and reflected photon). (B) We know an atom can absorb a photon – but can a free electron (in vacuum) adsorb a photon?

Problem 12.6 Every 2 years, more or less, *The New York Times* publishes an article in which some astronomer claims to have found an object traveling faster than the speed of light. Many of these reports result from a failure to distinguish what is *seen* from what is *observed*—that is, from a failure to account for light travel time. Here's an example: A star is traveling with speed v at an angle θ to the line of sight (Fig. 12.6). What is its apparent speed across the sky?

Problem 12.7 In a laboratory experiment a muon is observed to travel 800 m before disintegrating. A graduate student looks up the lifetime of a muon (2×10^{-6} s) and concludes that its speed was

$$v = \frac{800\,\text{m}}{2 \times 10^{-6}\,\text{s}} = 4 \times 10^{8}\,\text{m/s}.$$

Faster than light! Identify the student's error, and find the *actual* speed of this muon.



Problem 12.10 A sailboat is manufactured so that the mast leans at an angle $\bar{\theta}$ with respect to the deck. An observer standing on a dock sees the boat go by at speed v (Fig. 12.14). What angle does this *observer* say the mast makes?

Problem 12.17 Check Eq. 12.29, using Eq. 12.27. [This only proves the invariance of the scalar product for transformations along the x direction. But the scalar product is also invariant under *rotations*, since the first term is not affected at all, and the last three constitute the three-dimensional dot product $\mathbf{a}\cdot\mathbf{b}$. By a suitable rotation, the x direction can be aimed any way you please, so the four-dimensional scalar product is actually invariant under *arbitrary* Lorentz transformations.]

$$-\bar{a}^0\bar{b}^0 + \bar{a}^1\bar{b}^1 + \bar{a}^2\bar{b}^2 + \bar{a}^3\bar{b}^3 = -a^0b^0 + a^1b^1 + a^2b^2 + a^3b^3. \qquad (12.29)$$

$$\left.\begin{aligned} \bar{a}^0 &= \gamma(a^0 - \beta a^1), \\ \bar{a}^1 &= \gamma(a^1 - \beta a^0), \\ \bar{a}^2 &= a^2, \\ \bar{a}^3 &= a^3. \end{aligned}\right\} \qquad (12.27)$$

Problem 12.29 If a particle's kinetic energy is n times its rest energy, what is its speed?

Problem 12.30 Suppose you have a collection of particles, all moving in the x direction, with energies E_1, E_2, E_3, ...and momenta p_1, p_2, p_3, ... Find the velocity of the **center of momentum** frame, in which the total momentum is zero.